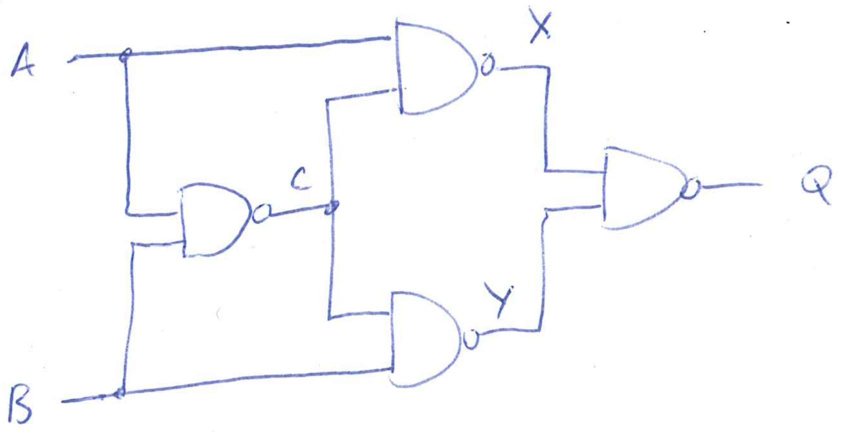
1. (a)



NAND

0	0	1
0	1	1
1	0	1
1	1	0

Truth table

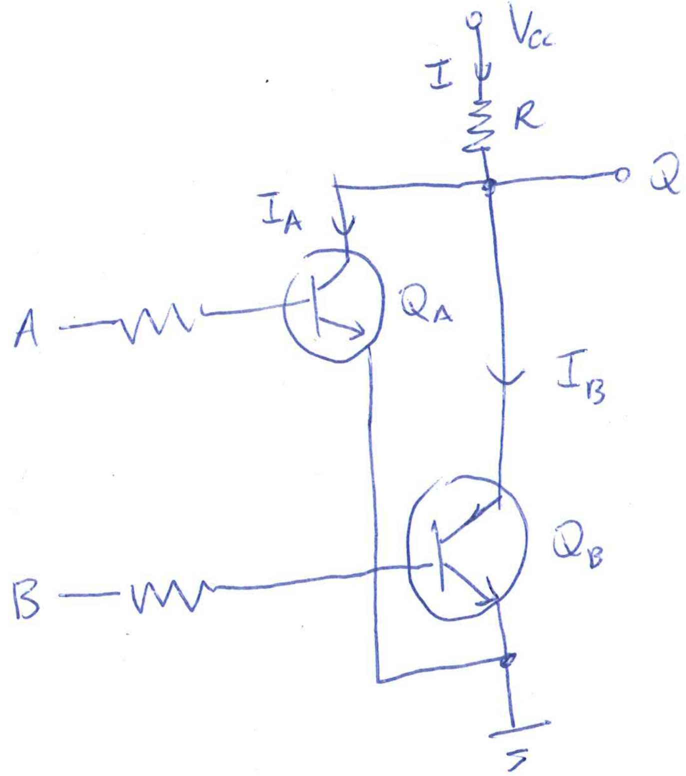
A	B	C	X	Y	Q
0	0	1	1	1	0
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	1	1	0

This is equivalent to the XOR truth table

$$Q = A \oplus B$$

2.5

1 (b)



$I = I_A + I_B$

Truth table

A	B	Q_A	Q_B	I_A	I_B	$Q = V_{cc} - IR$
0	0	~~ON~~ OFF	OFF	0	0	1 (Vcc)
0	1	OFF	ON	0	$\neq 0$	0
1	0	ON	OFF	$\neq 0$	0	0
1	1	ON	ON	$\neq 0$	$\neq 0$	0

(2.5) Equivalent to NOR truth table

$$Q = \overline{A + B}$$

2. (a)

A	B	$A \cdot B$	$\overline{A \cdot B}$	$\bar{A}$	$\bar{B}$	$\bar{A} + \bar{B}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

equivalent $\therefore \overline{A \cdot B} = \bar{A} + \bar{B}$

(1)

A	B	$A + B$	$\overline{A + B}$	$\bar{A}$	$\bar{B}$	$\bar{A} \cdot \bar{B}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

equivalent $\therefore \overline{A + B} = \bar{A} \cdot \bar{B}$

(1)

(b)

A	B	C	$A + B$	$(A+B) \cdot C$	$A \cdot C$	$B \cdot C$	$A \cdot C + B \cdot C$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	0	1	1
1	0	0	1	0	0	0	0
1	0	1	1	1	1	0	1
1	1	0	1	0	0	0	0
1	1	1	1	1	1	1	1

equivalent $\therefore (A+B) \cdot C = A \cdot C + B \cdot C$

(1)

(c)

A	$\bar{A}$	$A\cdot\bar{A}$
0	1	0
1	0	0

$\therefore A\cdot\bar{A} = 0 \quad \forall A.$

(1)

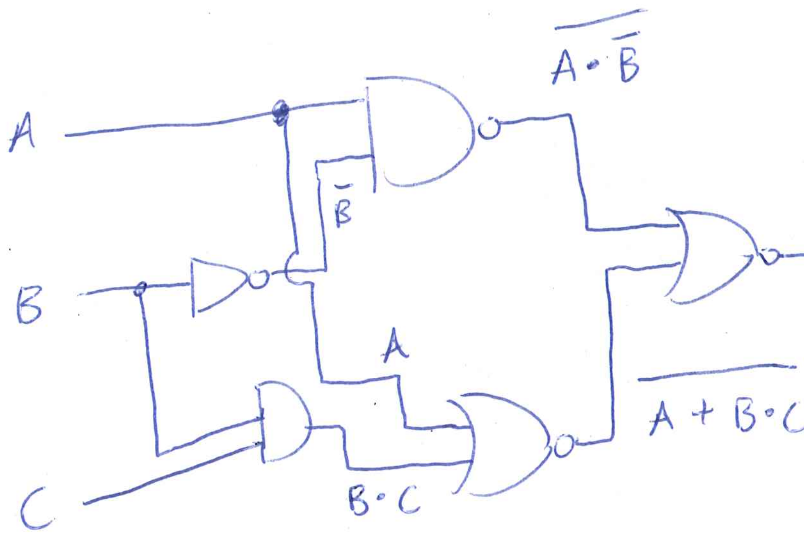
(d)

A	A	$A\cdot A$
0	0	0
1	1	1

$\therefore A\cdot A = A \quad \forall A$

(1)

3.



$Q = \overline{A\cdot\bar{B} + \overline{A+B\cdot C}}$

(1)

$$Q = \overline{\overline{A\cdot\overline{B}} + \overline{A+B\cdot C}}$$ (1)

use de Morgan's theorem $\overline{X+Y} = \overline{X}\cdot\overline{Y}$

$$= \left(\overline{\overline{A\cdot\overline{B}}}\right)\cdot\left(\overline{\overline{A+B\cdot C}}\right)$$ (1)

use $\overline{\overline{X}} = X$

$$= (A\cdot\overline{B})\cdot(A+B\cdot C)$$

use $(X+Y)\cdot Z = X\cdot Z + Y\cdot Z$

$$= A\cdot\overline{B}\cdot A + A\cdot\underbrace{\overline{B}\cdot B}_{0}\cdot C$$ (1)

use $\overline{B}\cdot B = 0$ & $X\cdot 0 = 0$

$$\therefore Q = A\cdot\overline{B}\cdot A$$

use $X\cdot Y\cdot Z = X\cdot Z\cdot Y$

proof

X	Y	Z	X·Y	X·Y·Z	X·Z	X·Z·Y
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	0	0	0
1	0	0	0	0	0	0
1	0	1	0	0	1	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1

(X·Y·Z and X·Z·Y columns: equivalent)

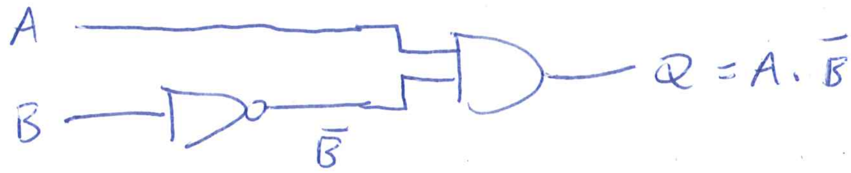
$$\therefore X\cdot Y\cdot Z = X\cdot Z\cdot Y$$

$$\therefore Q = A\cdot A\cdot\overline{B}$$ (1)

use $A\cdot A = A$

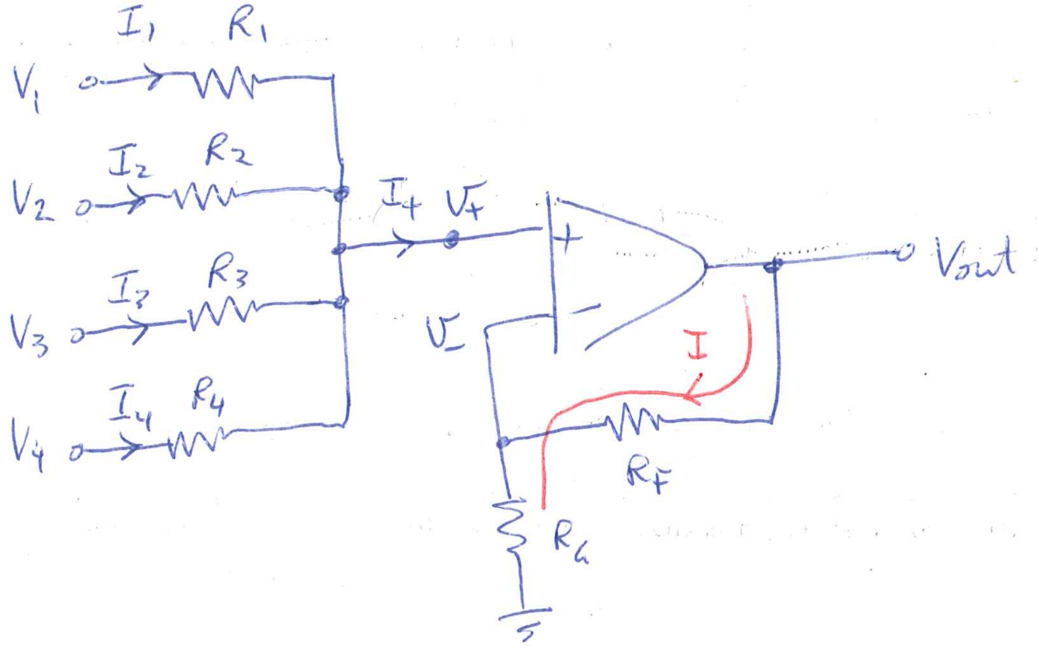
$$\boxed{\therefore Q = A\cdot\overline{B}}$$

∴ an equivalent circuit is



Note that input C has no effect whatsoever on the output Q.

4.



(a) start by finding V_-

$$V_{out} - I R_F - I R_G = 0$$

$$\therefore I = \frac{V_{out}}{R_F + R_G} \quad (1)$$

$$V_- = I R_G = V_{out} \frac{R_G}{R_F + R_G} \quad (1)$$

by Golden rule[2], know $\boxed{V_- = V_+ = V_{out} \frac{R_G}{R_F + R_G}}$

Find I_1 : $V_1 - I_1 R_1 = v_-$

(1)

$$\therefore I_1 = \frac{V_1 - v_-}{R_1} = \frac{1}{R_1}\left(V_1 - V_{out}\frac{R_G}{R_F + R_G}\right)$$

In the same way

$$I_j = \frac{1}{R_j}\left[V_j - V_{out}\frac{R_G}{R_F + R_G}\right]$$

where $j = 1, 2, 3, 4$.

By jcn rule $I_1 + I_2 + I_3 + I_4 = I_+$ (1)

but Golden Rule (#1) says $I_+ = 0$.

$$\therefore \sum_j I_j = 0$$

$$\therefore \sum_j \frac{1}{R_j}\left[V_j - V_{out}\frac{R_G}{R_F + R_G}\right] = 0 \quad (1)$$

distribut the sum

$$\sum_j \frac{V_j}{R_j} - V_{out}\frac{R_G}{R_F + R_G}\sum_j \frac{1}{R_j} = 0$$

Solve for V_{out}

$$V_{out} = \frac{\dfrac{R_F + R_G}{R_G} \displaystyle\sum_j \frac{V_j}{R_j}}{\displaystyle\sum_j \frac{1}{R_j}}$$

so finally:

$$V_{out} = \left(1 + \frac{R_F}{R_G}\right)\left[\sum_{j=1}^{4} \frac{1}{R_j}\right]^{-1} \sum_{j=1}^{4} \frac{V_j}{R_j}$$ (1)

(b) Take

$$\left.\begin{aligned} R_1 &= 10\,k\Omega \\ R_2 &= 5\,k\Omega \\ R_3 &= 2.5\,k\Omega \\ R_4 &= 1.25\,k\Omega \end{aligned}\right\} \left[\sum_j \frac{1}{R_j}\right]^{-1} = \frac{2}{3}\,k\Omega$$

$$\left.\begin{aligned} R_F &= 20\,k\Omega \\ R_G &= 10\,k\Omega \end{aligned}\right\} 1 + \frac{R_F}{R_G} = 3$$

$$\therefore V_{out} = (2\,k\Omega) \sum_{j=1}^{4} \frac{V_j}{R_j}$$

PHYS 231 Assign #6 Sol'ns (10)

V_4	V_3	V_2	V_1	$\sum \frac{V_j}{R_j}$	V_{out}
0	0	0	0	0	0
0	0	0	5	0.5 $\frac{V}{k\Omega}$	1 V
0	0	5	0	1	2
0	0	5	5	1.5	3
0	5	0	0	2	4
0	5	0	5	2.5	5
0	5	5	0	3	6
0	5	5	5	3.5	7
5	0	0	0	4	8
5	0	0	5	4.5	9
5	0	5	0	5	10
5	0	5	5	5.5	11
5	5	0	0	6	12
5	5	0	5	6.5	13
5	5	5	0	7	14
5	5	5	5	7.5 $\frac{V}{k\Omega}$	15 V

(2)

out of 8